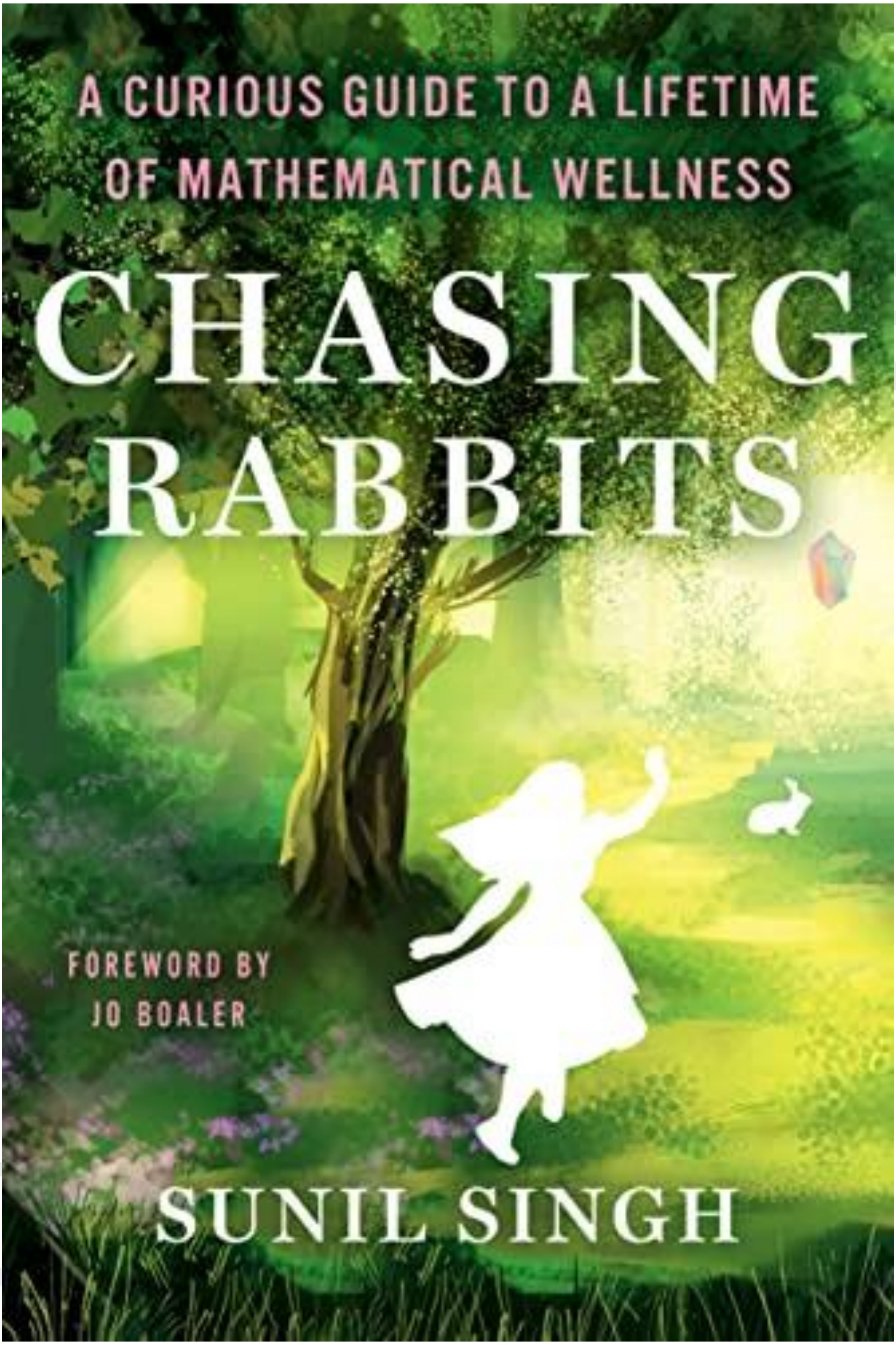
A CURIOUS GUIDE TO A LIFETIME
OF MATHEMATICAL WELLNESS
CHASING
RABBITS
FOREWORD BY
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A STORY OF LOST KNOWLEDGE

This story is about zero. To be specific, the *untold* story of zero. Well, geez, technically, that is not entirely true. The story was once told in its entirety by the Indian mathematicians and astronomers Aryabhata, Bhāskara, and Brahmagupta over a thousand years ago. Yes, the story was initially communicated through spoken poetry (remember writing materials like papyrus and bark would have been hard to come by). In my first book, the first chapter was dedicated to zero. However, while I detailed some of its colorful origins and mysticism, I was rather ignorant of its strange migration to the West through Islamic scholars. Remember the game Broken Telephone? This zero story is the almost fourteen-hundred-year-old version of that game—and it is still going! Just look at your typical QWERTY keyboard.

Do you see where zero is? Right at the very end, almost like an afterthought. While we may not begin our counting with zero —though we should—having it come after nine seems to symbolize the lack of understanding of what role zero is supposed to play in arithmetic. It was meant to be the central character in not only the laws of arithmetic, but in the laws of the universe! Instead, it became an understudy, a minor character, in one ridiculously confusing play of mathematics. Let's start at the beginning. But before we do that, let's reveal our storyteller: Jonathan Crabtree. When we tell stories to our friends, they often interlace with other stories or have a tendency to meander. With Crabtree, it is both!

Jonathan Crabtree is an Australian historian who has spent more than thirty years trying to recover the lost treasure of Indian mathematics—zero. It would not be a stretch to call him the Indiana Jones of ancient mathematics archivists. In fact, his own website (jonathancrabtree.com) makes it clear that this is the essence of his mission in life. But how did a person who earned an economics degree end up consumed by the rich history of Indian mathematics? Well, it has to do with a pivotal event in 1983, when Crabtree broke his back, and it was uncertain if he would ever walk again. His recovery soon became an inspiration. And, in 1987, a newspaper in Australia did a story on him. In the article he was quoted: "I hope to change the way the Western World teaches mathematics."

That goal has not changed, which is why I am sharing this story. Crabtree's passion to unravel the mystery of zero's deeper significance

—and its tragic omission from the general history of mathematics—is a burning light that should attract us all. Crabtree has weathered some significant losses in life—including the death of a child. It would have been understandable if the sum of these setbacks derailed him. In fact, it has been quite the opposite. Perhaps because the search and rescue of zero's lost history is a humanitarian mission. So let's travel back to seventh-century India and begin to unpack the story that Crabtree has been writing for more than three decades.

Zero was supposed to be the lowest number. That's it. Nothing was supposed to go below zero. How is walking seven feet to the left less than walking three feet to the right? How is it less than walking one foot to the right? Is the charge on an electron (−1) somehow less than the charge on a positron (+1)? Of course not. But these real-life situations are part of the problem of confusing students and teachers about the arithmetic that revolves around zero in terms of positive numbers and their opposite or negative numbers. I know. We are so ingrained with the traditional number line and looking at positive and negative numbers through the lens we grew up with that it is challenging to adopt a new model. Except this model is hardly new. It is incredibly old. It's the original one. It is the one that would make the most sense to younger children if this is what they saw first. Let us try to look at it through that perspective.

Do you really, really think that children were supposed to learn a problem like 1 − 2 *five years after* learning 2 − 1? The unwarranted separation of positive and negative numbers is the most glaring symptom that we merely mapped our historical misunderstanding of zero and negative numbers onto our math education, then called it a day.

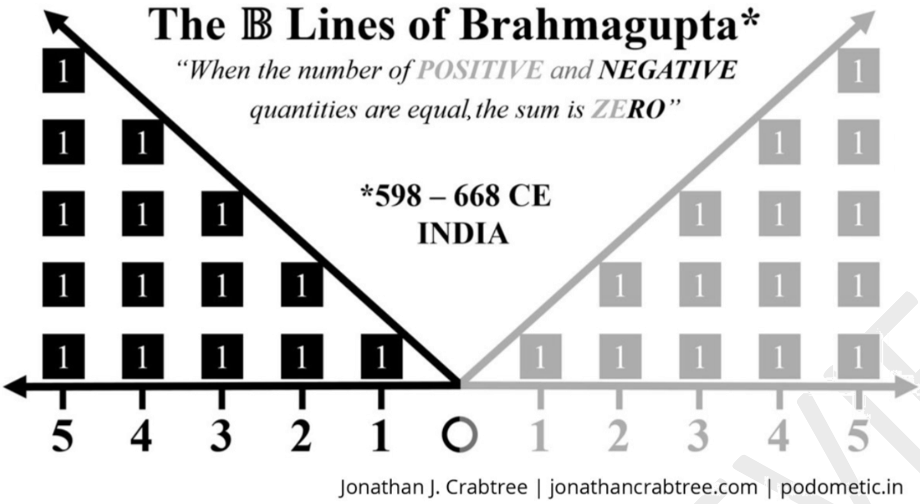
In 628 CE, Brahmagupta wrote his main work, *Brāhmasphutasiddhānta*. While a good chunk of it was astronomical in nature—and that is important—there were many key mathematical ideas, including the critical work surrounding zero. Here are Brahmagupta's 18 Sutras of Symmetry, as summarized by Crabtree. While many of these laws of addition, subtraction, multiplication, and division might seem like common sense, our final misunderstanding on the meaning of zero in mathematics, physics, and astronomy shows that we didn't do a great job extracting all that common sense contained within them:

- The sum of two positive quantities is positive.
- The sum of two negative quantities is negative.

- The sum of zero and a negative number is negative.
- The sum of zero and a positive number is positive.
- The sum of zero and zero is zero.
- The sum of a positive and a negative is their difference; or, if they are equal, zero.
- In subtraction, the less is to be taken from the greater, positive from positive.
- In subtraction, the less is to be taken from the greater, negative from negative.
- When the greater, however, is subtracted from the less, the difference is reversed.
- When positive is to be subtracted from negative, and negative from positive, they must be added together.
- The product of a negative quantity and a positive quantity is negative.
- The product of two negative quantities is positive.
- The product of two positive quantities is positive.
- Positive divided by positive or negative by negative is positive.
- Positive divided by negative is negative. Negative divided by positive is negative.
- Zero divided by a negative or positive number is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator.
- A positive or negative number when divided by zero is a fraction with zero as denominator.*
- Zero divided by zero is zero.*

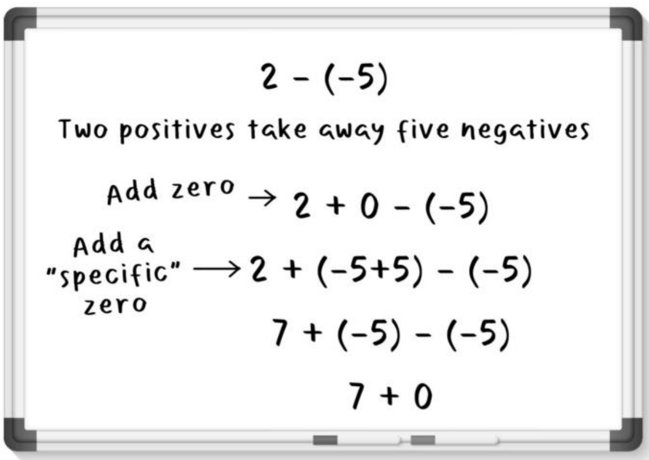
Note: **These laws don't align with modern mathematics, but they were the first attempt to define division by zero.*

A wonderful visual Crabtree created was to *bend* the number line at zero, so that it becomes literally the lowest point on a number line. The definition of zero that Brahmagupta intended is clear in the picture below—the *sum* of a positive and negative number of equal magnitudes is equal to zero. There is also a similar diagram to the correct interpretation of positive and negative values by he Chinese (the first appearance of zero in a Chinese text was in 1247.



We already do zero pairs in our classrooms, but we don't truly unleash that power to help students with questions involving negatives. In the question below, the quick standard is to instruct students to make the two minus signs a positive so that the question is now 2 + 5. While this achieves the correct answer, that is all that is achieved. There is no discussion as to the confusion that surely must lie in this situation. That confusion takes on a little clarity if we treat negatives and positives as objects. So now, it should be clear that it is currently impossible to do this question—we have 2 of one kind of object trying to subtract 5 of a different kind of object.

Adding zero as the next step might trigger a variety of responses, but the overwhelming one should be that adding zero does not change the question! And, finally, in the second to last line, we have *five negatives take away five negatives*. Same number of objects subtract same number of objects is always zero.



Let □ = $^{+}1$ and ■ = $^{-}1$
Two positives take away five negatives becomes
□□ take away ■ ■ ■ ■ ■, which can't be done! So we use Brahmagupta's Addition Sutra #5 and add a zero in the form of five positives and five negatives. Now we have □□ and □□□□□■ ■ ■ ■ ■ take away ■ ■ ■ ■ ■, which can be done, giving □□ and □□□□□ or seven positives. $^{+}2 - {}^{-}5 = {}^{+}7$.

So? What went wrong? What went squirrely? How did the mathematical world not take a *left at Albuquerque*? Crabtree's theory has to do with what the Arab mathematicians like al-Khwarizmi did with the twenty-four-chapter *Brāhmasphutasiddhānta*. Since much of the arithmetic and algebra works by these scholars were rooted in practical applications of inheritance and trading, it could be likely that the chapter on zero—more rooted in laws of the universe—was ignored. And since Brahmagupta's work on arithmetic, traveling along the Silk Road, would only make it to Western mathematicians via the Middle East, only what was heavily emphasized was going to be known. Zero's fate of being relegated to a placeholder was sealed by that drop of the mathematical baton.

So that is what Jonathan Crabtree has been doing for most of his life. Trying to find that baton and pass it off to the world—as it should have been over a thousand years ago. The best part of this story is that it is not over! I urge you to follow him at @jcrabtree on Twitter and spend many hours on his scholarly research that can be found on his website, podometic.in. He should be launching his children's books on *podometic* (a term he came up with to imply a new and improved arithmetic system) sometime in 2021. What better way to start the whole story of zero than with young children?